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Robust proportional-integral Kalman filter design using a convex optimization method

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Abstract

This paper proposes a design approach to the robust proportional-integral Kalman filter for stochastic linear systems under convex bounded parametric uncertainty, in which the filter has a proportional loop and an integral loop of the estimation error, providing a guaranteed minimum bound on the estimation error variance for all admissible uncertainties. The integral action is believed to increase steady-state estimation accuracy, improving robustness against uncertainties such as disturbances and modeling errors. In this study, the minimization problem of the upper bound of estimation error variance is converted into a convex optimization problem subject to linear matrix inequalities, and the proportional and the integral Kalman gains are optimally chosen by solving the problem. The estimation performance of the proposed filter is demonstrated through numerical examples and shows robustness against uncertainties, addressing the guaranteed performance in the mean square error sense.

Keywords: Proportional-integral observer; Kalman filter; Convex optimization; Robustness

1. Introduction

Unlike proportional observers such as the Luenberger observer [1] and the Kalman filter [2], which have only a proportional loop of the estimation error, proportional-integral (PI) observers have an additional integral feedback loop, addressing that it can increase steady-state estimation accuracy and improve estimation robustness against disturbances, unknown inputs, or modeling errors in a similar manner to the proportional-integral control scheme. This benefit comes from the additional degrees of freedom in the observer design. This type of the observer scheme was first introduced by Wojciechowski [3] for single-input single-output systems and then extended to multivariable systems by Kaczorek [4] and Shafai and Carroll [5], addressing a more flexible way for performing design objectives. For example, this loop can be used to identify unknown inputs or nonlinearities [6], to increase the stability margin in the LTR design [7, 8], to attenuate or decouple disturbances [9, 10], or to detect sensor or actuator faults [9, 11].

Several design methods for these PI observers have been introduced: pole-placement methods [6, 7], eigenstructure assignments [12], H-infinity norm minimization problems [9, 11], or minimum estimation error variance approaches [13, 14]. For instance, Duan et al. [12] introduced a parametric eigenstructure assignment method that provides complete degrees of freedom in designing PI observers, which leads to lower eigenvalue sensitivities. Mark et al. [11] adopted the minimization problem of the H-infinity norm of the transfer function from the disturbance to the estimation error in order to make disturbances insensitive in estimating faults or states.

From the point of view of minimizing the estimation error variance, Baş et al. [14] introduced a PI Kalman filter where the proportional and integral Kalman gains were obtained from the Riccati equa-

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tion, leading to minimum error variance. To deal with transient performance and to improve the stability margin, a fading coefficient and an integral effect coefficient were suggested as additional design parameters. However, since these parameters are highly dependent on the nature of the system, the nature of disturbances and system requirements, they have to be tuned in a trial and error for better performance. Further, this design approach requires knowledge of the exact system matrices.

This paper proposes an LMI-based robust proportional-integral Kalman filter for stochastic linear systems under convex bounded uncertainty. The minimization problem of the upper bound on the estimation error variance for all admissible parametric uncertainties is formulated as a convex optimization problem. The proportional and the integral gains are optimally chosen by solving the convex optimization problem subject to linear matrix inequalities, leading to a minimum bound on the estimation error variance. The proposed design methodology can easily handle structured uncertainties in state, input, or output matrices.

The rest of this paper is organized as follows. Section 2 introduces the structure of the proportional-integral Kalman filter. Section 3 proposes a new design approach for the filter using a convex optimization problem. In Section 4, the estimation performance of the filter is demonstrated through numerical examples.

2. Problem formulation

Consider the following linear stochastic system with parametric uncertainties:

$$\begin{aligned} \dot{x} &= Ax + Bw \\ y &= Cx + Dw \end{aligned} \tag{1}$$

where $x \in \Re^{n\times 1}$ and $y \in \Re^{p\times 1}$ denote the state vector and the output vector, respectively. The noise vector, w, includes the process noise and the measurement noise that are assumed to be white noise with the identity power spectral density. The system matrix defined as

$$M := \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

is unknown but belongs to a given convex bounded polyhedral domain Ω described by

$$\Omega := \left\{ M : M = \sum_{i=1}^{m} \lambda_i M_i ; \quad \lambda_i \geq 0, \quad \sum_{i=1}^{m} \lambda_i = 1 \right\},$$

which means any admissible system matrix M can be written as an unknown convex combination of m given extreme matrices such that

$$M_i = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}$$

where A_i, B_i, C_i, and D_i, $i = 1, \ldots , m$, are given matrices [15]. In the case of $m = 1$, the system matrix M is perfectly known. The mathematical description of the convex bounded domain is sufficiently general in practice. For example, when only some elements of M are unknown, this case can be easily expressed by a proper choice of the set of extreme matrices [16].

For the system of Eq. (1), a proportional-integral (PI) observer can be designed as follows:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + L(y - C\hat{x}) + Kz \\ \dot{z} &= -\alpha z + H(y - C\hat{x}) \end{aligned} \tag{2}$$

where the matrices L and K represent a proportional gain and an integral gain, respectively. The variable z is related to the weighted integral of the output estimation error. The constant value α stands for a fading effect coefficient that regulates the transient response. The matrix H is an integral effect coefficient that improves the stability margin [14]. The two design parameters are assumed to be preselected by designers. The error dynamic equation of the state vector is obtained by subtracting Eq. (2) from Eq. (1) as described by

$$\dot{e} = (A - LC)e + (B - LD)w - Kz \tag{3}$$

where $e := x - \hat{x}$ denotes the state estimation error. The state estimation error can be rewritten as an output of the following augmented system by connecting Eq. (3) to Eq. (2) as follows:

$$\begin{aligned} \dot{x}_a &= A_a x_a + B_a w \\ e &= C_a x_a \end{aligned} \tag{4}$$

where

$$x_a := \begin{bmatrix} e \\ z \end{bmatrix}, \quad A_a := \begin{bmatrix} A - LC & -K \\ HC & -\alpha I \end{bmatrix}$$

$$B_a := \begin{bmatrix} B - LD \\ HD \end{bmatrix}, \quad C_a := \begin{bmatrix} I & 0 \end{bmatrix}.$$

The state estimation error variance, as time goes to infinity, is given by

$$
\begin{aligned}
\mathbf{E}[e^T e] &= tr\mathbf{E}[ee^T] \\
&= tr[C_a X C_a^T]
\end{aligned}
\tag{5}
$$

where tr and $\mathbf{E}$ denote *trace* and *expectation*, respectively, and the matrix X is defined as the solution of the following Lyapunov equation:

$$
A_a X + X A_a^T + B_a B_a^T = 0 \tag{6}
$$

where X is a symmetric non-negative definite matrix. Analogously, the estimation error variance is rewritten as

$$
\mathbf{E}[e^T e] = tr[B_a^T X B_a] \tag{7}
$$

where X satisfies the following Lyapunov equation [15]:

$$
A_a^T X + X A_a + C_a^T C_a = 0\,. \tag{8}
$$

Note that A_a is stable if $X > 0$ satisfies Eq. (8) and the pair (A_a, C_a) is observable. Since the solution of the Lyapunov equation has a monotonicity property, there exists $P > X$ satisfying

$$
\begin{aligned}
&Minimize \quad tr[B_a^T P B_a] \\
&subject\ to \quad A_a^T P + P A_a + C_a^T C_a < 0
\end{aligned}
\tag{9}
$$

where P is a symmetric positive definite solution, which leads to the error dynamics in Eq. (4) is asymptotically stable. The optimization problem in Eq. (9) can be reformulated as Eq. (10) by introducing a new variable matrix N and by using the Schur complement [17]:

$$
\begin{aligned}
&Minimize \quad tr[N] \\
&subject\ to \quad P = P^T > 0 \\
&\qquad \begin{bmatrix} A_a^T P + P A_a & C_a^T \\ C_a & -I \end{bmatrix} < 0 \\
&\qquad \begin{bmatrix} N & B_a^T P \\ P B_a & P \end{bmatrix} > 0
\end{aligned}
\tag{10}
$$

where N, P, L, and K are design variables. Here the value of the objective function will be greater than the estimation error variance defined in Eq. (7). However, by minimizing the objective function, the optimal solution in Eq. (10) will be arbitrarily close to the solution of the Lyapunov equation and the gap between the minimum value of Eq. (10) and the estimation error variance will be arbitrarily small.

The PI Kalman filter can be designed by solving Eq. (10), which offers the proportional and the integral Kalman gains defined in Eq. (2). The problem is that the constraints in Eq. (10) are nonlinear matrix inequalities. This makes the observer design difficult in obtaining the optimal solution. Our purpose is to convert the two inequalities into corresponding linear matrix inequalities.

3. New formulation for a proportional-integral Kalman filter design

This section deals with a PI Kalman filter design for convex bounded parametric uncertain systems. A convex optimization problem subject to linear matrix inequalities is introduced in order to design the PI Kalman filter.

Theorem 1: Assume the system matrices, A, B, C, and D, are known. For given design parameters, H, α and ρ, the proportional-integral Kalman filter of the form of Eq. (2) can be designed if the following convex optimization problem is feasible:

$$
\begin{aligned}
&Minimize \quad tr[N] \\
&subject\ to \\
&\begin{bmatrix} N & B^T P_1 - D^T S_1^T + D^T H^T P_2^T & B^T P_1 - D^T S_1^T + D^T H^T S_2^T \\ P_1 B - S_1 D + P_2 H D & P_1 & P_1 \\ P_1 B - S_1 D + S_2 H D & P_1 & S_4 \end{bmatrix} > 0 \\
&\begin{bmatrix} A^T P_1 + P_1 A - C^T S_1^T - S_1 C + C^T H^T P_2^T + P_2 H C & A^T P_1 - C^T S_1^T + C^T H^T S_2^T - S_3 - \alpha P_1 & I \\ -S_3^T + P_1 A - S_1 C + S_2 H C - \alpha P_1 & -S_3^T - S_3 - 2\alpha S_4 & 0 \\ I & 0 & -I \end{bmatrix} < 0 \\
&\begin{bmatrix} S_4 & S_2 \\ S_2^T & \rho I \end{bmatrix} > 0, \;\; P_1 = P_1^T > 0, \;\; S_4 = S_4^T > 0
\end{aligned}
\tag{11}
$$

where N, P_1, P_2, S_1, S_2, S_3, and S_4 are variables. With the optimal solutions that minimize the state estimation error variance, the resulting proportional and integral Kalman gains are calculated as follows:

$$
L = P_1^{-1} S_1\,, \quad K = P_1^{-1} S_3 P_1^{-1} P_2\,, \tag{12}
$$

where P_1 is a symmetric positive definite matrix and

P_2 is a nonsingular matrix.

Proof: Partition P with suitable dimension according to A_a, as described by

$$P := \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix} \tag{13}$$

where P_1 and P_3 are $n\times n$ symmetric positive definite matrices [15]. It is assumed that the matrix P_2 is nonsingular. By substituting the definition for the matrices in Eqs. (4) and (13) into them, the two inequalities in Eq. (10) are rewritten as Eqs. (14) and (15), respectively:

$$\begin{bmatrix} A_a^T P + PA_a & C_a^T \\ C_a & -I \end{bmatrix} = \left[\begin{array}{c|c|c} A^T P_1 + P_1 A - C^T L^T P_1 - P_1 LC + C^T H^T P_2^T + P_2 HC & A^T P_2 - C^T L^T P_2 + C^T H^T P_3 - P_1 K - \alpha P_2 & I \\ \hline -K^T P_1 + P_2^T A - P_2^T LC + P_3 HC - \alpha P_2^T & -K^T P_2 - P_2^T K - 2\alpha P_3 & 0 \\ \hline I & 0 & -I \end{array}\right] < 0 \tag{14}$$

and

$$\begin{bmatrix} N & B_a^T P \\ PB_a & P \end{bmatrix} = \left[\begin{array}{c|c|c} N & B^T P_1 - D^T L^T P_1 + D^T H^T P_2^T & B^T P_2 - D^T L^T P_2 + D^T H^T P_3 \\ \hline P_1 B - P_1 LD + P_2 HD & P_1 & P_2 \\ \hline P_2^T B - P_2^T LD + P_3 HD & P_2^T & P_3 \end{array}\right] > 0 \tag{15}$$

where P_1, P_2, P_3, L, K, and N are design variables.

These nonlinear matrix inequalities can be converted into linear matrix inequalities through the change of coordinate transformation [15]. Premultiplying and postmultiplying Eq. (14) by T_1^T and T_1, respectively, where

$$T_1 = \begin{bmatrix} I & 0 & 0 \\ 0 & P_2^{-1}P_1 & 0 \\ 0 & 0 & I \end{bmatrix}, \tag{16}$$

and introducing the new variables such as

$$S_1 := P_1 L, \qquad S_2 := P_1 P_2^{-T} P_3, \tag{17}$$
$$S_3 := P_1 K P_2^{-1} P_1, \quad S_4 := P_1 P_2^{-T} P_3 P_2^{-1} P_1,$$

Eq. (14) can be rewritten as follows:

$$\left[\begin{array}{c|c|c} A^T P_1 + P_1 A - C^T S_1^T - S_1 C + C^T H^T P_2^T + P_2 HC & A^T P_1 - C^T S_1^T + C^T H^T S_2^T - S_3 - \alpha P_1 & I \\ \hline -S_3^T + P_1 A - S_1 C + S_2 HC - \alpha P_1 & -S_3^T - S_3 - 2\alpha S_4 & 0 \\ \hline I & 0 & -I \end{array}\right] < 0 \tag{18}$$

where $P^{-T} := (P^{-1})^T$. Here, the variables S_2 and S_4 are dependent with the relation of

$$S_4 = S_2 P_3^{-1} S_2^T. \tag{19}$$

However, since P_3 is a positive definite matrix, there exists an arbitrary scalar ε such that

$$S_4 > \varepsilon S_2 S_2^T. \tag{20}$$

Using the Schur complement [17], Eq. (20) can be converted into a linear matrix inequality as follows:

$$\begin{bmatrix} S_4 & S_2 \\ S_2^T & \rho I \end{bmatrix} > 0 \tag{21}$$

where $\rho = 1/\varepsilon$. Now, the two matrix inequalities of Eqs. (18) and (21) are linear with respect to the independent variables P_1, P_2, S_1, S_2, S_3, and S_4. The nonlinear matrix inequality of Eq. (14) is then converted into the two linear matrix inequalities of Eqs. (18) and (21).

In a similar manner, Eq. (15) can be reformulated to an LMI. Premultiplying and postmultiplying Eq. (15) by T_2^T and T_2, respectively, where

$$T_2 = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & P_2^{-1}P_1 \end{bmatrix}, \tag{22}$$

Eq. (15) is rewritten as

$$\left[\begin{array}{c|c|c} N & B^T P_1 - D^T L^T P_1 + D^T H^T P_2^T & B^T P_1 - D^T L^T P_1 + D^T H^T P_3 P_2^{-1} P_1 \\ \hline P_1 B - P_1 LD + P_2 HD & P_1 & P_1 \\ \hline P_1 B - P_1 LD + P_1 P_2^{-T} P_3 HD & P_1 & P_1 P_2^{-T} P_3 P_2^{-1} P_1 \end{array}\right] > 0 \tag{23}$$

and furthermore, by introducing Eq. (17), Eq. (23) is given by

$$\left[\begin{array}{c|c|c} N & B^T P_1 - D^T S_1^T + D^T H^T P_2^T & B^T P_1 - D^T S_1^T + D^T H^T S_2^T \\ \hline P_1 B - S_1 D + P_2 HD & P_1 & P_1 \\ \hline P_1 B - S_1 D + S_2 HD & P_1 & S_4 \end{array}\right] > 0 \tag{24}$$

Eq. (24) is an LMI with the variables, N, P_1, P_2, S_1, S_2, and S_4. Thus, the nonlinear matrix inequality of Eq. (15) is converted into the linear matrix inequality of Eq. (24). Replacing the two matrix inequalities in Eq. (10) with the three LMIs of Eqs. (18), (21), and (24), the nonconvex optimization problem in Eq. (10) is converted into the convex optimization problem of Eq. (11). With the optimal values of the variables S_1 and S_3 and the relation of Eq. (17), the proportional and the integral Kalman gains are calculated as Eq. (12).

Now, consider a robust PI Kalman filter for the uncertain system Eq. (1). For all the admissible parameter uncertainties, a robust filter with a guaranteed performance in the mean square error sense has an upper bound of the estimation error variance. That is, the optimal value of the objective function of Eq. (10) provides a minimum upper bound for the estimation error variance. In the case of m = 1, *i.e.*, for the exactly known system matrix, the bound coincides with the estimation error variance. Corollary 1 offers a robust PI Kalman filter that minimizes the upper bound on the estimation error variance for all the admissible convex bounded parametric uncertainties.

Corollary 1: For the convex bounded uncertain system defined in Eq. (1) and for given design parameters, H, α, and ρ, a robust proportional-integral Kalman filter can be designed if the following convex optimization problem is solvable:

Minimize $tr[N]$

subject to

$$\left[\begin{array}{c|c|c} N & B_i^T P_1 - D_i^T S_1^T + D_i^T H^T P_2^T & B_i^T P_1 - D_i^T S_1^T + D_i^T H^T S_2^T \\ \hline P_1 B_i - S_1 D_i + P_2 H D_i & P_1 & P_1 \\ \hline P_1 B_i - S_1 D_i + S_2 H D_i & P_1 & S_4 \end{array}\right] > 0$$

$$\left[\begin{array}{c|c|c} A_i^T P_1 + P_1 A_i - C_i^T S_1^T - S_1 C_i + C_i^T H^T P_2^T + P_2 H C_i & A_i^T P_1 - C_i^T S_1^T + C_i^T H^T S_2^T - S_3 - \alpha P_1 & I \\ \hline -S_3^T + P_1 A_i - S_1 C_i + S_2 H C_i - \alpha P_1 & -S_3^T - S_3 - 2\alpha S_4 & 0 \\ \hline I & 0 & -I \end{array}\right] < 0$$

$$\begin{bmatrix} S_4 & S_2 \\ S_2^T & \rho I \end{bmatrix} > 0,\ P_1 = P_1^T > 0,\ S_4 = S_4^T > 0,\ i = 1, \cdots, m \tag{25}$$

where $\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}$ is the m vertices of the polytope Ω.

The design variables in the optimization problem are N, P_1, P_2, S_1, S_2, and S_4. From the optimal solutions of Eq. (25), the resulting proportional and integral Kalman gains are calculated as the same as Eq. (12).

Remark: The optimization problem in Corollary 1 belongs to a convex optimization problem, which means the objective and the constraints are both convex. Thus, the final solution is uniquely determined as a global one.

For unknown systems with parametric uncertainties, the structure of the PI observer defined in Eq. (2) can be replaced with Eq. (26), in which the nominal system matrices are used as follows:

$$\begin{aligned} \dot{\hat{x}} &= A_n \hat{x} + L(y - C_n \hat{x}) + Kz \\ \dot{z} &= -\alpha z + H(y - C_n \hat{x}) \end{aligned} \tag{26}$$

where the matrices A_n and C_n are the nominal matrices of A and C, respectively. Since the nominal system lies within the polyhedral domain Ω, the effect on the estimation performance due to the gap between the pair (A, C) and the pair (A_n, C_n) can be minimized by the proposed minimization problem, addressing a guaranteed bound in the mean square error sense.

4. Examples

In this section, numerical examples are given in order to demonstrate the performance of the robust proportional-integral Kalman filter proposed in this study. Optimization problems involving LMI formulations can be solved numerically and effectively with interior-point methods. Some advantages and some commercial software in regard with the problem are found in a reference book [17]. In this paper, the convex optimization problem in Corollary 1 is implemented in MATLAB by using the LMI Lab package [18].

Example 1:

Consider a second-order time-varying uncertain stochastic system:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -2 + 0.3\Delta & -1 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} w \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0.1 \end{bmatrix} w \end{aligned}$$

where Δ is an unknown parameter satisfying $|\Delta(t)| \le 1$, and $\Delta = 0$ corresponds to the nominal system. In this example, the uncertainty is assumed to be $\Delta(t) = sin(t)$. This norm-bounded set falls to the polyhedral convex set with the two extreme matrices M_1 and M_2. The noise vector, w, includes both the process noise and the measurement noise; *i.e.*, $w = [w_1^T\ w_2^T]^T$, where w_1 and w_2 denote the process noise and the measurement noise, respectively. It is assumed that w_1 and w_2 are white noise with the identity power spectral density, respectively.

The performance of the proposed PI Kalman filter is demonstrated by comparing it with that of a robust Kalman filter suggested by de Souza and Trofino [15]. The robust Kalman filter with only a proportional Kalman gain provides a guaranteed estimation error variance under polytope uncertainties and is designed from an optimization problem subject to linear matrix inequalities. By solving the optimization problem proposed by de Souza and Trofino [15], the robust Kalman filter is designed as follows:

$$\dot{\hat{x}} = \begin{bmatrix} -1.16 & 0.73 \\ -8.39 & -8.98 \end{bmatrix} \hat{x} + \begin{bmatrix} -1.34 \\ 0.50 \end{bmatrix} y$$

where the eigenvalues of the observer matrix are $\{-2.04, -8.10\}$.

In designing the proposed PI Kalman filter, the design parameters shown in Corollary 1 are chosen as follows: $H = [0.1 \quad 0.1]^T$, $\alpha = 0.1$, and $\rho = 10$. The resulting optimal gains are calculated as

$$L = \begin{bmatrix} 10.76 \\ 7.48 \end{bmatrix}, \quad K = \begin{bmatrix} 689.69 & -693.94 \\ 611.64 & -614.83 \end{bmatrix}.$$

Each optimal objective value in the two filters, which means the minimum upper bound of the estimation error variance for all the admissible uncertainties, is 0.198 for the proposed PI Kalman filter and 0.239 for the robust Kalman filter [15], respectively. This shows that under the parametric uncertainties, the proposed PI Kalman filter offers a lower bound on the estimation error variance than the robust Kalman filter [15] in the given problem.

The estimated states are illustrated and compared in Fig. 1. The proposed observer quickly converges to the true values in spite of parametric uncertainties, while the robust Kalman filter is slow and provides large estimation errors as shown in Fig. 1.

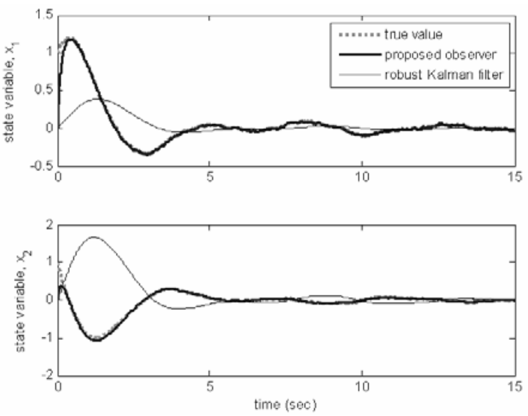

Fig. 1. Comparison of the estimation performance in Example 1.

Example 2:

A laser bonder is a soldering machine that connects the leads to an integrated circuit chip with pads on the IC board. The bonder head moves up and down by a linear voice coil motor, melting the solder and completing the connection. The position sensor measures the vertical position of the head. The information on the head velocity is useful to control the soldering machine accurately [19].

A third-order plant model for a laser bonder is expressed as follows [19]:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ -221+\Delta & 0 & 6.70 \\ 0 & -5.69 & -2.65 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0.44 \end{bmatrix} V_m + \begin{bmatrix} 0 & 0 \\ 10 & 0 \\ 1 & 0 \end{bmatrix} w$$

$$y = [1 \quad 0 \quad 0]x + [0 \quad 0.1]w$$

where the state vector, x, includes the vertical position of the head, the head velocity, and the driving current, in order. The input variable, V_m is the applied voltage and the output variable, y, is the head position. In this paper, the applied voltage is assumed to be zero in order to design the robust Kalman filter formulated

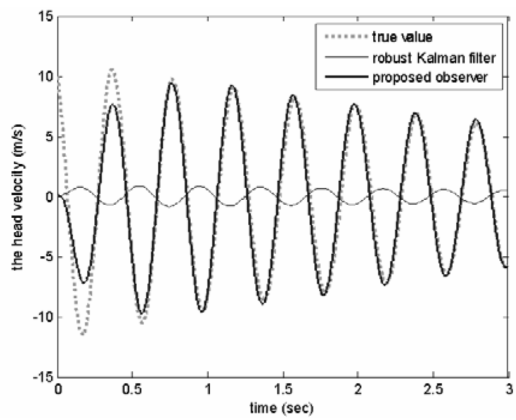

Fig. 2. Comparison of the estimated head velocity in Example 2.

based on no input signal [15]. The noise vector, w, includes the process noise and the measurement noise of zero mean Gaussian with the identity power spectral density. The parametric uncertainty, Δ, which comes from the change of the spring coefficient, varies within 10 % of its nominal value, *i.e.*, $|\Delta(t)| \le 22$. In this study, it is assumed to be $\Delta(t) = 22\sin(t)$.

In designing the proposed filter, the design parameters shown in Corollary 1 are chosen as follows: $H = [0.1 \quad 0.1 \quad 0.1]^T$, $\alpha = 0.1$, and $\rho = 10$. The resulting optimal gains are calculated as

$$L = \begin{bmatrix} 7.28 \\ 26.49 \\ -35.37 \end{bmatrix}, \quad K = \begin{bmatrix} 5.64 & -2.82 & -2.82 \\ 18.04 & -7.88 & -7.88 \\ -34.54 & -17.50 & 17.50 \end{bmatrix}.$$

The robust Kalman filter [15] is designed as given by

$$\dot{\hat{x}} = \begin{bmatrix} 4.88 & 241.69 & -45.19 \\ -1.34 & -15.19 & 3.73 \\ -0.15 & -8.85 & -11.17 \end{bmatrix} \hat{x} + \begin{bmatrix} -150.74 \\ 1.81 \\ 4.64 \end{bmatrix} y$$

where the eigenvalues of the observer matrix are $\{-11.56, -4.96 \pm j15.83\}$.

The estimated velocity of the head is illustrated and compared in Fig. 2. The robust PI Kalman filter proposed in this paper offers little steady-state estimation error, while the robust Kalman filter [15] shows a large estimation error. This indicates that the integral action is effective in improving the steady-state accuracy in designing a robust and minimum estimation error variance filter.

5. Conclusions

This paper provides a design approach of a robust proportional-integral Kalman filter for convex bounded parametric uncertain stochastic systems. The proportional and the integral Kalman gains are optimally chosen by solving a convex optimization problem subject to linear matrix inequalities, which offers a minimum guaranteed bound on the estimation error variance for all the admissible convex bounded uncertainties. Simulation results demonstrate that the robust PI Kalman filter offers the minimum upper bound of the estimation error variance and warrants robustness against both stochastic and deterministic uncertainties. In particular, the robustness of the proposed PI Kalman filter is guaranteed against structured uncertainty.

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